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# Massive dualities in six dimensions

K Behrndt<sup>1</sup>, E Bergshoeff<sup>2</sup>, D Roest<sup>2</sup> and P Sundell<sup>3</sup>

<sup>1</sup> Institut für Physik, Humboldt University, Invalidenstraße 110, 10115 Berlin, Germany

<sup>2</sup> Institute for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

<sup>3</sup> Department of Theoretical Physics, Uppsala University, Box 803, S-75108 Uppsala, Sweden

E-mail: behrndt@physik.hu-berlin.de, e.a.bergshoeff@phys.rug.nl, d.roest@phys.rug.nl  
and per.sundell@teorfys.uu.se

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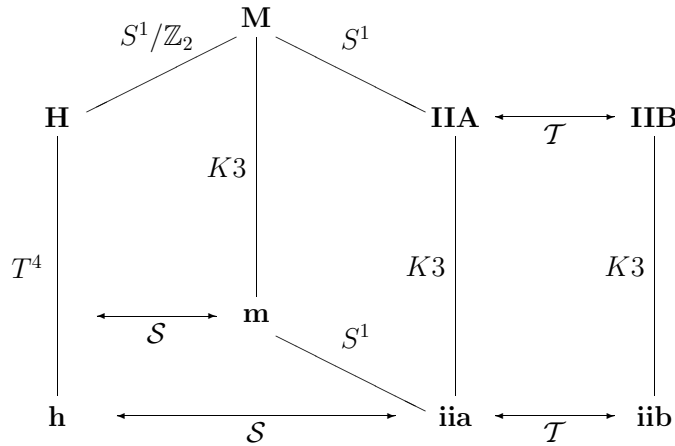
## Abstract

We study compactifications of string theory and M-theory to six dimensions with background fluxes. The non-zero fluxes lead to additional mass parameters. We derive the S- and T-duality rules for the corresponding (massive) supergravity theories. Specifically, we investigate the massive T-duality between Type IIA superstring theory compactified on K3 with background fluxes and Type IIB superstring theory compactified on K3. Furthermore, we generalize to the massive case the 6D ‘string–string’ S-duality between M-theory on  $K3 \times S^1$  and the heterotic string on  $T^4$ . Whereas in the case of massive T-duality the mass parameters are in the fundamental representation of the U-duality group  $O(4, 20)$  we find that in the case of massive S-duality they are in the 3-index antisymmetric representation. In the latter case, the mass parameters involved extend those of Kaloper and Myers. We apply our duality rules to massive brane solutions, such as the domain wall solutions corresponding to the mass parameters and find new massive brane solutions. Finally, we discuss the higher dimensional interpretation of the dualities and brane solutions.

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## 1. Introduction

It is well known that M-theory and the five string theories are related via different T- and S-dualities. In the low-energy limit, this manifests itself by relations between the corresponding supergravities or  $S^1$ -compactifications thereof. When considering compactifications to six dimensions other dualities arise. For instance, Type IIA and Type IIB string theory compactified on K3 and the heterotic string theory compactified on  $T^4$  are related by various dualities [1–3]. At the level of the corresponding  $D = 6$ ,  $N = 2$  supergravities the explicit



**Figure 1.** Web of dualities. The  $S^1$  reduction of M-theory to the Type IIA superstring may be seen as an S-duality since the strong coupling limit of the Type IIA superstring corresponds to a large radius limit of the  $S^1$ . The  $S^1/\mathbb{Z}_2$  reduction of M-theory to the heterotic model is the Horava–Witten scenario [5]. In the lower line h, iia and iib are six-dimensional theories whereas m is seven-dimensional. The higher dimensional origin is indicated by vertical lines. The duality relations between the theories are indicated by horizontal two-sided arrows with  $S$  denoting an S-duality and  $T$  indicating a T-duality.

forms of the different duality relations were derived in [4]. The relations are illustrated in figure 1.

In this paper, we consider an extension of the results of [4] where we apply, instead of ordinary Kaluza–Klein reductions, Scherk–Schwarz reductions or, more generally, compactifications with non-zero background fluxes. This allows us to extend the six-dimensional dualities to duality relations involving a set of mass parameters  $\{m\}$ . The new rules constitute a massive deformation of the old rules in the sense that for  $\{m = 0\}$  we recover the duality rules of [4].

Compactifications with background fluxes have attracted renewed interest in view of the fact that they lead to massive supergravities which play an important role in (i) the AdS/CFT correspondence [6], (ii) the Randall–Sundrum scenario [7, 8] and (iii) recent cosmological applications (see e.g. [9, 10]). These types of compactifications are a generalization of Scherk–Schwarz (SS) reductions [11] where a non-trivial background flux is given to some field strength  $F_{p+1}$  of a  $p$ -form tensor field  $A_p$ . The non-zero values are taken in the directions of a non-trivial  $(p + 1)$ -cycle of the compactification manifold.

The case  $p = 0$  where  $A_0$  is the R–R axionic scalar of IIB supergravity has been considered in [12] and generalized in [13, 14]. Similarly, the NS–NS axionic scalars of the heterotic model were used in [15] to generate massive deformations of its toroidal reductions. In all these cases, the same deformations can be obtained directly in the lower dimensional theory by the gauging of a subgroup of the U-duality group. The mass parameters are the structure constants of the gauged subgroup in some basis [16, 17]. More recently, the SS reduction of massive IIA supergravity [18] on K3 with background fluxes has been considered [19]. The resulting massive supergravity theory in six dimensions does not seem to correspond to the gauging of any global symmetry.

In generalized SS reductions the following subtlety plays an important role. Consider the reduction on a product manifold  $A \times B$ . For an ordinary Kaluza–Klein (KK) reduction the order of compactification on  $A$  and  $B$  is of no influence. First compactify on  $A$  and next on

$B$  or the other way around gives the same (massless) supergravities. It is said that two KK reductions commute with each other. The situation changes for SS reductions [20]. Here the order does matter in general, i.e. reducing over an internal manifold  $A \times B$  gives a different result than when reducing over  $B \times A$ . The different ways of performing SS reductions lead to supergravities with different massive deformations. Therefore *SS reductions do not commute* in general.

The dependence on the reduction scheme arises since one throws away Fourier modes on the internal manifolds. In a KK reduction all but the zero-mode is thrown away, in which case the order of compactification is irrelevant. On the other hand, in a SS reduction one keeps a combination of the zero-mode and higher Fourier modes, depending on the mass parameter. In such a case, the order in which one throws away the higher order Fourier modes is relevant and leads to different results in general.

A natural question is whether the massive deformations of the supergravities preserve the duality relations. The T-duality between Type IIA and Type IIB string theory in  $D = 10$  has been generalized to the massive case involving a single mass parameter [12]. However, the 11-dimensional origin of this mass parameter is not well understood. Due to this one cannot construct a massive S-duality between massive IIA and  $D = 11$  supergravity. Recently, the T-duality between massive IIA supergravity compactified on K3 [19] and IIB supergravity compactified on K3 (see figure 1) has been extended to the massive case, as derived by [21] and, independently, by the present authors.

It has been found in the SS reduction of Type IIA string theory on K3 [19] that the mass parameters fill multiplets of the U-duality group, which is perturbative in this case. Similarly, it has been found that in SS reductions of Type IIA string theory on  $T^2$  and  $T^4$  [22], the mass parameters fill multiplets of the perturbative part of the U-duality group. In this sense, the non-perturbative part of the U-duality group is broken, while the perturbative part survives as a so-called pseudo-symmetry, i.e. the Lagrangian is invariant provided we also transform the mass parameters.

The prevailing theme seems to be that perturbative dualities can be extended to the massive case but that for non-perturbative dualities this is problematic. In this paper, we will consider massive extensions of the duality relations in six dimensions. Indeed, the T-duality between Type IIA and Type IIB string theory compactified on K3 can be generalized to include mass parameters, while the S-duality between Type IIA string theory compactified on K3 and the heterotic model on  $T^4$  is more subtle, as argued in [19, 21]. Remarkably, we are able to construct a massive S-duality in six dimensions by considering SS reductions in a different order. An intriguing feature is that the massive S-duality contains a set of mass parameters that are in the 3-index antisymmetric representation of the U-duality group  $O(4, 20)$  whereas the massive T-duality involves a set of mass parameters that are in the fundamental representation of  $O(4, 20)$ . It remains an open issue whether there are massive dualities for *all* massive deformations occurring in compactifications of string theory and/or M-theory.

This paper is organized as follows. In section 2, we derive the massive T-duality rules between massive Type IIA and Type IIB string theory on K3. The same duality rules have, independently, been derived in [21]. Furthermore, we apply the massive T-duality rules to massive brane solutions. Next, in section 3, we construct massive S-duality rules between M-theory compactified on  $K3 \times S^1$  and the heterotic string theory compactified on  $T^4$ . Again we apply these massive S-duality rules to brane solutions. In section 4, we consider the higher dimensional interpretation of the 1/2 BPS brane solutions in  $D = 6$ . In particular, we discuss the domain wall solutions and their relations under duality. Finally, we will conclude with a discussion and summary section. Our notational conventions are given in appendix A. In appendix B, we will consider the SS reduction of the heterotic string theory on  $T^4$ . This result

will be needed in order to construct the massive S-duality rules in section 3 and generalizes the work of [15].

## 2. Massive T-duality

In this section we consider the extension to the massive case of the T-duality in six dimensions between Type IIA and Type IIB string theory compactified on K3. The contents of this section overlap with [21] where a similar analysis was performed.

### 2.1. Supergravity relations

The Type IIA theory, compactified on K3 to six dimensions, has the following massless field content:

$$\text{IIA: } \{g, B, \phi, V^A, M^{AB}\}. \quad (1)$$

Here  $g$  is the metric,  $B$  is the NS–NS 2-form and  $\phi$  is the dilaton. The  $A, B$  indices ( $A, B = 1, \dots, 24$ ) refer to the U-duality group  $O(4, 20)$ . The 1-forms  $V^A$  are in the fundamental representation of  $O(4, 20)$ . The scalar matrix  $M^{AB}$  parametrizes the coset  $O(4, 20)/(O(4) \times O(20))$ . It contains  $80 = 4$  (dilaton) +  $76$  (axionic) scalars. Being an element of  $O(4, 20)$  it satisfies

$$M_{AD}^{-1} = L_{AB} M^{BC} L_{CD}. \quad (2)$$

The  $O(4, 20)$  metric  $L_{AB}$  is iteratively defined by

$$L_{AB} = L^{AB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & L_{ab} \end{pmatrix} \quad (3)$$

where  $L_{ab}$  ( $a, b = 1, \dots, 22$ ) is the  $O(3, 19)$  metric. Similarly, the scalar matrix  $M^{AB}$  is iteratively defined by

$$M^{AB} = \begin{pmatrix} -e^{-2\sigma} + \ell_a \ell_b M^{ab} - \frac{1}{4} e^{2\sigma} \ell^4 & -\frac{1}{2} e^{2\sigma} \ell^2 & \ell_a M^{ab} - \frac{1}{2} e^{2\sigma} \ell^2 \ell^b \\ -\frac{1}{2} e^{2\sigma} \ell^2 & -e^{2\sigma} & e^{2\sigma} \ell^b \\ M^{ab} \ell_b - \frac{1}{2} e^{2\sigma} \ell^2 \ell^a & e^{2\sigma} \ell^a & M^{ab} - e^{2\sigma} \ell^a \ell^b \end{pmatrix} \quad (4)$$

while  $M^{kl} = L^{kl} = (\mathbb{I}_{16})^{kl}$  with  $k, l = 1, \dots, 16$ . Here the  $\ell_a$  are 22 axionic scalars ( $\ell^2 = \ell_a \ell^a$ ),  $\sigma$  is a dilaton scalar and  $M^{ab}$  parametrizes the coset  $O(3, 19)/(O(3) \times O(19))$ . The  $O(4, 20)$ -duality symmetry acts as

$$V^A \rightarrow \Omega_B^A V^B \quad M^{AB} \rightarrow \Omega_C^A \Omega_D^B M^{CD} \quad (5)$$

with  $\Omega_B^A \in O(4, 20)$ . This group consists of 276 generators, of which 76 shift the axions with a constant:  $\ell \rightarrow \ell + m$ .

In a recent paper, the Scherk–Schwarz generalized reduction of ten-dimensional massive IIA supergravity [18] on K3 was performed [19]. This compactification yields the Lagrangian  $\mathcal{L}_{\text{IIA},6} = \frac{1}{2} \sqrt{-g} e^{-2\phi} [-R + 4|d\phi|^2 - \frac{3}{4}|dB|^2 + \frac{1}{8} \text{Tr}(dM dM^{-1}) - e^{2\phi}|F^A|^2 - 2e^{2\phi}|m^A|^2]$

$$- \frac{1}{8} \epsilon [BF^A F_A - m^A B^2 F_A + \frac{1}{3} m^2 B^3] \quad (6)$$

with the field strength

$$F^A \equiv dV^A + m^A B. \quad (7)$$

We use a notation where the  $O(4, 20)$ -indices in  $|F^A|^2$  and  $|m^A|^2$  are contracted using the scalar matrix  $M^{-1}$ , e.g.

$$|m^A|^2 = m^A (M^{-1})_{AB} m^B. \quad (8)$$

The Scherk–Schwarz reduction introduces a total of 24 mass parameters  $m^A$ . Note that the NS–NS 2-form  $B$  becomes massive in the presence of fluxes. Furthermore, the non-zero  $m^A$  break the  $O(4, 20)$  symmetry to a pseudo-symmetry [19]:

$$V^A \rightarrow \Omega_B^A V^B \quad M^{AB} \rightarrow \Omega_C^A \Omega_D^B M^{CD} \quad m^A \rightarrow \Omega_B^A m^B \quad (9)$$

with  $\Omega_B^A \in O(4, 20)$ . The 24 mass parameters fill the fundamental multiplet  $m^A$ .

We wish to compare the 6D IIA Lagrangian with the 6D IIB Lagrangian that results from the KK compactification of IIB supergravity on K3. In order to do so, it is convenient to bring the 6D IIB Lagrangian into a more convenient form. In the fluxless case, the six-dimensional IIB action can be written in a manifest  $O(5, 21)$  invariant form. The model contains 5 self-dual and 21 anti-self-dual 2-forms with corresponding duality relations. To match the IIA field content it is convenient to extract one self-dual and one anti-self-dual 2-form from the total of 26. This gives us the unconstrained NS–NS 2-form  $B$  plus 4 self-dual and 20 anti-self-dual 2-forms. The 105 scalars of the theory are organized into a dilaton  $\phi$ , 24 axionic scalars  $\ell^A$  while the remaining 80 scalars parametrize a  $O(4, 20)/(O(4) \times O(20))$  coset matrix  $M^{AB}$ . Thus the field content consists of

$$\text{IIB: } \{g, B, \phi, B^A, \ell^A, M^{AB}\} \quad (10)$$

with the 24 2-forms  $B^A$  being either self-dual or anti-self-dual. There is no generalized SS reduction of IIB supergravity on K3 since the IIB theory has only field strengths of odd rank while the manifold K3 has only harmonic forms of even rank. Therefore, the six-dimensional IIB pseudo-action [23] has no mass parameters and reads

$$\begin{aligned} \mathcal{L}_{\text{IIB},6} = & \frac{1}{2} \sqrt{-g} e^{-2\phi} \left[ -R + 4|d\phi|^2 - \frac{3}{4}|dB|^2 + \frac{1}{8} \text{Tr}(dM dM^{-1}) \right. \\ & \left. + \frac{1}{2} e^{2\phi} |d\ell^A|^2 + \frac{3}{8} e^{2\phi} |H^A|^2 \right] - \frac{1}{16} \epsilon [\ell_A dB dB^A] \end{aligned} \quad (11)$$

with the field strengths and the (anti-)self-duality relations

$$H^A \equiv dB^A + \ell^A dB \quad H^A = L^{AB} M_{BC}^{-1*} H^C. \quad (12)$$

The action (11) has the following axionic shift symmetries

$$\ell^A \rightarrow \ell^A + m^A \quad B^A \rightarrow B^A - m^A B \quad (13)$$

with  $m^A$  constant. We will use this symmetry below to introduce masses when reducing to five dimensions<sup>4</sup>.

To establish T-duality between the IIA and IIB theories an isometry is required. As in the fluxless case, we will reduce both IIA and IIB actions to five dimensions. The IIA reduction formulae read (expressing 6D fields in terms of 5D fields)

$$\begin{aligned} \text{IIA} \quad & \text{5D} \\ g_{\underline{z}\underline{z}} = & -e^{-2\phi-2\sigma/\sqrt{3}} \\ g_{\underline{z}\mu} = & -e^{-2\phi-2\sigma/\sqrt{3}} A_\mu \\ g_{\mu\nu} = & e^{-2\phi+2\sigma/\sqrt{3}} g_{\mu\nu} - e^{-2\phi-2\sigma/\sqrt{3}} A_\mu A_\nu \\ B_{\mu\nu} = & (B + AC)_{\mu\nu} \\ B_{\underline{z}\mu} = & C_\mu \\ \phi = & -\phi + \frac{1}{\sqrt{3}} \sigma \end{aligned} \quad (14)$$

<sup>4</sup> By using the full  $O(5, 21)$  U-duality symmetry one can induce more mass parameters in five dimensions, as discussed in appendix B. In this section, we only use the axionic shift symmetries since this suffices for the derivation of the massive T-duality rules.

$$\begin{aligned} V_\mu^A &= V_\mu^A + \ell^A A_\mu \\ V_{\underline{z}}^A &= \ell^A \\ M^{AB} &= M^{AB} \end{aligned}$$

where we use the notation  $(AC)_{\mu\nu} = A_{[\mu} C_{\nu]}$ . These reduction rules are identical to the massless case, i.e. they correspond to ordinary Kaluza–Klein reductions. There is no dependence on the internal coordinate  $\underline{z}$ . Instead, the IIB reduction formulae read, again expressing 6D fields in terms of 5D fields,

$$\begin{aligned} \text{iib} \quad & \underline{\text{5D}} \\ g_{\underline{z}\underline{z}} &= -e^{2\phi+2\sigma/\sqrt{3}} \\ g_{\underline{z}\mu} &= -e^{2\phi+2\sigma/\sqrt{3}} C_\mu \\ g_{\mu\nu} &= e^{-2\phi+2\sigma/\sqrt{3}} g_{\mu\nu} - e^{2\phi+2\sigma/\sqrt{3}} C_\mu C_\nu \\ B_{\mu\nu} &= (B + C A)_{\mu\nu} \\ B_{\underline{z}\mu} &= A_\mu \\ B_{\mu\nu}^A &= (B^A + C V^A)_{\mu\nu} - 2m^A (B + C A)_{\mu\nu} \underline{z} \\ B_{\underline{z}\mu}^A &= V_\mu^A \\ \phi &= \frac{2}{\sqrt{3}} \sigma \\ \ell^A &= \ell^A + 2m^A \underline{z} \\ M^{AB} &= M^{AB}. \end{aligned} \tag{15}$$

Note the linear dependence on the internal coordinate  $\underline{z}$ , which takes the form of a  $\underline{z}$ -dependent axionic shift symmetry (13). This particular  $\underline{z}$ -dependence will introduce mass parameters in the five-dimensional theory. We are dealing here with a Scherk–Schwarz reduction similar to that used in [12]. For consistency in five dimensions, the shift symmetry (13) is crucial; it implies that the reduced theory is independent of the internal coordinate  $\underline{z}$ . After dimensional reduction, the (anti-)self-dual 2-forms split up into 1- and 2-forms. The duality relations can be used to eliminate one of the two forms. For obvious reasons, we will keep the 24 vectors  $V^A$  and eliminate the 2-forms  $B^A$ . Reducing the above six-dimensional actions in this way, we find that both the massive IIA and the IIB theory yield the following massive five-dimensional  $N = 2$  theory:

$$\begin{aligned} \mathcal{L}_{\text{II},5} &= \frac{1}{2} \sqrt{g} e^{-2\phi} \left[ -R + 4|\text{d}\phi|^2 - \frac{4}{3}|\text{d}\sigma|^2 - \frac{3}{4} e^{4(\phi-\sigma/\sqrt{3})} |H|^2 + \frac{1}{8} \text{Tr}(\text{d}M \text{d}M^{-1}) - |F^A|^2 \right. \\ &\quad + \frac{1}{2} e^{4\sigma/\sqrt{3}} |G^A|^2 - 2e^{-4(\phi-\sigma/\sqrt{3})} |m^A|^2 + e^{4\phi} |\text{d}C|^2 + e^{-4\sigma/\sqrt{3}} |\text{d}A|^2 \Big] \\ &\quad + \frac{1}{4} \epsilon [C F^A F_A + (B + C A) F^A G_A + 2m^A B C F_A - \frac{1}{2} m^A (B + C A)^2 \text{d}\ell_A] \end{aligned} \tag{16}$$

with the field strengths

$$H = \text{d}B + C \text{d}A + A \text{d}C \quad F^A = \text{d}V^A + \ell^A \text{d}A + m^A (B + C A) \quad G^A = \text{d}\ell^A - 2m^A C. \tag{17}$$

Note that the mass parameters  $m^A$  break the  $O(5, 21)$  symmetry to an  $O(5, 21)$  pseudo-symmetry. An  $O(4, 20)$  subgroup acts as

$$V^A \rightarrow \Omega_B^A V^B \quad M^{AB} \rightarrow \Omega_C^A \Omega_D^B M^{CD} \quad m^A \rightarrow \Omega_B^A m^B \quad \ell^A \rightarrow \Omega_B^A \ell^B \tag{18}$$

with  $\Omega_B^A \in O(4, 20)$ . The fields  $C$  and  $B$  are massive, as could be expected from their six-dimensional IIA origin.

Having the relations (14) and (15) between six- and five-dimensional fields at our disposal, we can use these to relate fields in six dimensions to one isometry. We thus derive the following massive T-duality relations between Type IIA and Type IIB string theory compactified on  $K3$ :

$$\begin{aligned}
& \text{iib} \quad \text{iia} \\
& \phi = \phi - \frac{1}{2} \log(-g_{\underline{z}\underline{z}}) \\
& g_{\underline{z}\underline{z}} = 1/g_{\underline{z}\underline{z}} \\
& g_{\underline{z}\mu} = B_{\underline{z}\mu}/g_{\underline{z}\underline{z}} \\
& g_{\mu\nu} = g_{\mu\nu} - (g_{\underline{z}\mu}g_{\underline{z}\nu} - B_{\underline{z}\mu}B_{\underline{z}\nu})/g_{\underline{z}\underline{z}} \\
& B_{\underline{z}\mu} = g_{\underline{z}\mu}/g_{\underline{z}\underline{z}} \\
& B_{\mu\nu} = B_{\mu\nu} - (g_{\underline{z}\mu}B_{\underline{z}\nu} - g_{\underline{z}\nu}B_{\underline{z}\mu})/g_{\underline{z}\underline{z}} \\
& B_{\underline{z}\mu}^A = V_{\mu}^A - V_{\underline{z}}^A g_{\underline{z}\mu}/g_{\underline{z}\underline{z}} \\
& \ell^A = V_{\underline{z}}^A + 2m^A \underline{z} \\
& M^{AB} = M^{AB}
\end{aligned} \tag{19}$$

where on the left-hand side are the IIB fields and on the right-hand side are the IIA fields. Note that the duality transformations of  $B_{\mu\nu}^A$  are not given. Their duality rules are encoded in the reduced self-duality conditions relating IIA and IIB field strengths:

$$\frac{3}{2} H^A = -e^{2\phi} M^{AB} \star F_B. \tag{20}$$

In comparison with the massless case, only the T-duality rules for  $\ell^A$  and  $H^A$  receive massive corrections. The latter receives corrections due to the fact that the curvature  $F^B$  in the above duality relation receives massive corrections.

This finishes our derivation of the massive T-duality rules.

## 2.2. Brane solutions

Our starting point is the set of basic dp-brane solutions<sup>5</sup> ( $p = 0, 1, 2, 3, 4$ ) that preserves  $1/2$  supersymmetry. The general dp-brane solution is given by [24]

$$\begin{aligned}
ds_{dp}^2 &= \Omega^{-1} [dt^2 - ds_p^2] - \Omega dx_{5-p}^2 & e^{2\phi} &= \Omega^{1-p} \\
F_{01\dots pm}^A &= \partial_m H^A & \mathcal{M}^{AB} &= \mathbb{I}^{AB} + 2\Omega^{-1} H^A H^B
\end{aligned} \tag{21}$$

where  $F^A$  is a  $(p+2)$ -form field strength and  $\Omega^2 \equiv H_A H^A$ . The functions  $H^A$  are harmonic in the  $(5-p)$ -dimensional transverse space. The dp-branes can carry 24 different charges  $\forall p$ :  $1 \times 2 = 2$  come from  $Dp \perp D(p+4)$  and  $3 \times 2 + 16 \times 1 = 22$  from  $D(p+2) \perp D(p+2)$  in terms of ten-dimensional intersections<sup>6</sup>. All these intersections have an obvious 11D interpretation in terms of M-theory compactified on  $S^1 \times K3$ , i.e. first compactifying on  $S^1$  and then on  $K3$ . For  $p = 0, 2, 4$  we obtain  $M5//W1$ ,  $M2//KK6$  and  $KK6//KK6$ , respectively. Here it is understood that the special isometry of the  $KK6$ -branes is in the  $S^1$ -direction.

We note that the dp-branes with  $p = 1, 3$  are solutions of the iib theory which has no mass parameters. The other dp-branes, with  $p = 0, 2, 4$ , are solutions of the iia theory which can have massive deformations. Only the construction of the d4-brane solution requires the

<sup>5</sup> To indicate the higher dimensional origin of the six-dimensional solutions we use lower-case letters in the six-dimensional case and higher-case letters in the ten- or eleven-dimensional case. In section 4, one can find the precise correspondence between the lower and higher dimensional branes.

<sup>6</sup> The total number of independent charges can also be deduced by counting the independent ways in which one can obtain a non-zero expression for  $m^A m^B L_{AB}$  thereby using the fact that  $L_{AB}$  has a  $8 \times 8$  off-diagonal piece and a  $16 \times 16$  diagonal piece. Thus one obtains  $4 \times 2 + 16 \times 1 = 24$  independent charges.



presence of a mass parameter, as can be seen from the expression for the harmonic functions  $H^A$ :

$$H^A = 1 + m^A x \quad (22)$$

where  $x$  is the single transverse direction. To stress this fact we will call this solution the md4-brane (massive d4-brane). The d0-brane and d2-brane solutions (21) are only valid in the massless case, i.e. in the absence of any mass parameters. To relate the d3-brane and md4-brane solutions one must use the massive T-duality rules (19). This is the natural replacement of the 10D massive T-duality rules of [12]. The reason that the massive T-duality works is that the d3-brane solution, assuming an extra isometry direction in the two-dimensional transverse space, has the proper (linear)  $\underline{z}$ -dependence, i.e. a linear axion.

The massive T-duality rules cannot be used to generate a massive md2-brane solution out of the d3-brane. The reason is that in this case the d3-brane must be reduced over a worldvolume isometry direction without any linear  $\underline{z}$ -dependence. Hence the reduction does not introduce any mass parameters to relate to a massive md2-brane solution. Instead, one can T-dualize the d3-brane to a *massless* d2-brane solution. Quite generally, one can only generate a massive iia solution out of a massless iib solution if the iib solution has the correct  $\underline{z}$ -dependence, i.e. a linear axion.

Alternatively, we can apply the massive T-duality rules and construct a new iib solution, with a linear axion, out of a given massive iia solution. We note that the iia theory is a compactification of the massive IIA supergravity theory for which massive BPS brane solutions preserving 1/4 supersymmetry are known. The simplest solution to consider is a Kaluza–Klein compactification of the massive (fundamental) string [25, 26]. This yields the following  $D = 6$  massive fundamental string or mfl<sub>iia</sub>-solution

$$\begin{aligned} ds_{iia}^2 &= H^{-1}[dt^2 - dz^2] - dx_4^2 & e^{-2\phi} &= H & V_t &= mz \\ B_{tz} &= H^{-1} + 1 & H &= mz - \sum_{r=1}^4 \frac{m_r^2}{4} x_r^2 + H'(x). \end{aligned} \quad (23)$$

Here  $m$  and  $V$  denote the  $A = 8$  component of  $m^A$  and  $V^A$ , respectively. The function  $H'(x)$  is a harmonic function in the four transverse directions  $x_r$  and  $\sum_r m_r^2 = m^2$ . The constants  $m_r$  are the integration constants. For  $m = 0$ , the mfl<sub>iia</sub>-solution reduces to the fundamental string or fl<sub>iia</sub>-solution. The mfl<sub>iia</sub>-solution can be viewed as an intersection of a fl-string ( $B_{tz} \neq 0$ ), a d4-brane ( $m \neq 0$ ) and a d0-brane ( $V \neq 0$ ).

Note that  $H$  depends on the coordinate  $z$  in the string direction. Therefore, unlike in the  $m = 0$  case, the string direction no longer represents an isometry and we cannot T-dualize this direction to obtain a wave solution of the iib theory. Interestingly enough, there is a related iib wave solution [27] which does not break any supersymmetry. After T-dualizing the iib wave solution of [27] backs to the iia side, the resulting metric and antisymmetric tensor are the same as for the mfl<sub>iia</sub>-solution if one ignores the linear  $z$ -dependence. Although this T-dual solution still solves the equations of motion, supersymmetry is broken [27].

Another possibility is to assume an isometry in one of the four transverse directions, say  $x$ , and to T-dualize in this direction. An application of the massive T-duality rules leads to a massive mfl<sub>iib</sub>-solution with a linear axion:

$$\begin{aligned} ds_{iib}^2 &= H^{-1}[dt^2 - dz^2] - dx_4^2 & e^{-2\phi} &= H & V_t &= mz \\ B_{tz} &= H^{-1} + 1 & H &= mz - \sum_{r=1}^4 \frac{m_r^2}{4} x_r^2 + H'(x) \\ \ell &= 2mx & C_{tx} &= -mz. \end{aligned} \quad (24)$$

Here  $\ell$  and  $C_{tx}$  denote the  $A = 8$  component of  $\ell^A$  and  $B_{tx}^A$ , respectively. The new mfl<sub>iib</sub>-solution can be viewed as an intersection of a f1-string ( $B_{tz} \neq 0$ ), a d3-brane ( $\ell \neq 0$ ) and a d1-brane ( $C_{tx} \neq 0$ ).

### 3. Massive S-duality

#### 3.1. Supergravity relations

We now discuss the massive S-duality. In the massless case, the S-duality we consider relates the heterotic string theory compactified on  $T^4$  to the Type IIA string theory compactified on  $K^3$ . The explicit S-duality rules read (with heterotic fields on the left-hand and iia fields on the right-hand side)

$$\begin{aligned} \underline{\text{h}} \quad & \underline{\text{iia}} \\ g_{\mu\nu} &= e^{-2\phi} g_{\mu\nu} \\ \phi &= -\phi \\ \tilde{H} &= e^{-2\phi} \star H. \end{aligned} \tag{25}$$

Here we have used the definitions  $\tilde{H} \equiv d\tilde{B} + V_A dV^A$  and  $H = dB$ .

We have not been able to construct a massive S-duality between the heterotic string theory on  $T^4$  and Type IIA string theory compactified on  $K^3$ . As has already been explained in [19, 21], one of the obstructions to construct a massive deformation of (25) is that the NS 2-form at the IIA side, see equation (7), becomes massive. At first sight, one can therefore not apply a dualization procedure for the 2-form in the Lagrangian. The situation is similar to the massive 2-form of Roman's supergravity. From that case it is known that one can perform the duality (25) also on massive 2-forms provided we dualize the pair

$$\{B(\text{massive}), m_A V^A(\text{massless})\} \rightarrow \{\tilde{B}(\text{massless}), m_A \tilde{V}^A(\text{massive})\}. \tag{26}$$

Such a massive S-duality can only be defined at the level of the field equations [28]. This leads to the following massive S-duality rules:

$$\begin{aligned} \underline{\text{h}} \quad & \underline{\text{iia}} \\ g_{\mu\nu} &= e^{-2\phi} g_{\mu\nu} \\ \phi &= -\phi \\ \tilde{H} &= e^{-2\phi} \star H \\ m_A \tilde{F}^A &= m_A M^{AB} \star F_B \end{aligned} \tag{27}$$

with  $m_A \tilde{F}^A$  a 4-form curvature. The different (massive) curvatures are given by

$$\begin{aligned} \tilde{H} &= d\tilde{B} + V_A dV^A + 2m_A V^A B - 2m_A \tilde{V}^A \\ m_A \tilde{F}^A &= m_A d\tilde{V}^A - m_A V^A H + \frac{1}{2} m^2 B^2 \\ H &= dB \quad F_A = dV_A + m_A B \end{aligned} \tag{28}$$

where  $m_A \tilde{V}^A$  is a 3-form potential. The fact that the S-dual version of the IIA theory (6) does not allow an action shows that it has no obvious higher dimensional origin. Indeed, we have not been able to obtain the S-dual theory via some generalized compactification of the 10D heterotic theory.

However, we are able to construct a massive S-duality between the heterotic string theory compactified on  $T^4$  and M-theory compactified on  $K^3 \times S^1$ . Our construction is based on the observation that the heterotic string theory compactified on  $T^3$  is S-dual to M-theory compactified on  $K^3$  [1, 2]. To construct the massive S-duality we apply a SS reduction of

both seven-dimensional theories on a circle  $S^1$  to six dimensions. This is in the spirit of the 4D massive dualities discussed in [29].

We first discuss the massless S-duality in seven dimensions between the heterotic string theory compactified on  $T^3$  and M-theory on  $K3$ . The field contents of the dual theories are

$$\text{heterotic on } T^3: \{g, \tilde{B}, \phi, V^a, M_{ab}^{-1}\} \quad \text{M-theory on } K3: \{g, C^{(3)}, \phi, V^a, M_{ab}^{-1}\} \quad (29)$$

where the scalar indices  $a, b$  run from  $1, \dots, 22$ . Note that the field content between the two theories only differs in the sense that the heterotic string on  $T^3$  has a 2-form  $\tilde{B}$  while M-theory on  $K3$  contains a (dual) 3-form  $C^{(3)}$ . All other fields take the same form. The scalar matrix  $M_{ab}^{-1}$  parametrizes the coset  $O(3, 19)/(O(3) \times O(19))$ . It contains 57 scalars, of which 54 are axionic and 3 dilatonic. The 22 vectors  $V^a$  fall into the fundamental representation of the U-duality group. The corresponding massless actions read as

$$\begin{aligned} \mathcal{L}_{\text{het},7} &= \frac{1}{2}\sqrt{g} e^{-2\phi} \left[ -R + 4|d\phi|^2 - \frac{3}{4} |d\tilde{B} - V_a dV^a|^2 + \frac{1}{8} \text{Tr}(dM dM^{-1}) - |dV^a|^2 \right] \\ \mathcal{L}_{\text{m},7} &= \frac{1}{2}\sqrt{g} e^{-2\phi} \left[ -R + 4|d\phi|^2 + \frac{1}{3} e^{4\phi/5} |dC^{(3)}|^2 + \frac{1}{8} \text{Tr}(dM dM^{-1}) - e^{8\phi/5} |dV^a|^2 \right] \\ &\quad + \frac{1}{24} \epsilon C^{(3)} dV_a dV^a. \end{aligned} \quad (30)$$

Both have a global  $O(3, 19)$  U-duality symmetry

$$V^a \rightarrow \Omega^a_b V^b \quad M^{ab} \rightarrow \Omega^a_c \Omega^b_d M^{cd} \quad (31)$$

with  $\Omega^a_b \in O(3, 19)$ . The two theories in 7D are related by the (massless) S-duality

$$\begin{aligned} \underline{7D \text{ h}} \quad \underline{\text{m}} \\ g_{\mu\nu} &= e^{-8\phi/5} g_{\mu\nu} \\ \phi &= -\phi \\ \frac{3}{4} \star (d\tilde{B} - V_a dV^a) &= e^{-2\phi} (dC^{(3)}) \end{aligned} \quad (32)$$

with 7D heterotic fields on the left-hand side and M-theory fields on the right-hand side. Note that the third relation interchanges Bianchi identities and equations of motion. Under this S-duality the Wess–Zumino term of the M-theory is interchanged with the Chern–Simons term in the 3-form field strength of the heterotic theory.

We now perform a SS reduction on both seven-dimensional theories using the results of appendix B. The reduction rules for the heterotic fields are given in appendix B (see equation (57)). The reduction rules of the 7D M-theory fields are

$$\begin{aligned} \underline{\text{m}} \quad \underline{\text{ii a}} \\ g_{zz} &= -e^{8\phi/5 - 4\sqrt{3}\sigma/5} \\ g_{z\mu} &= -e^{8\phi/5 - 4\sqrt{3}\sigma/5} V_\mu^\bullet \\ g_{\mu\nu} &= e^{-2\phi/5 + 8\sqrt{3}\sigma/15} g_{\mu\nu} - e^{8\phi/5 - 4\sqrt{3}\sigma/5} V_\mu^\bullet V_\nu^\bullet \\ C_{\mu\nu\rho}^{(3)} &= C_{\mu\nu\rho}^{(3)} \\ C_{\mu\nu\bar{z}}^{(3)} &= B_{\mu\nu} \\ \phi &= \phi + \frac{1}{\sqrt{3}}\sigma \\ V_\mu^a &= \Omega^a_b(z) (V_\mu^b + \ell^b V_\mu^\bullet) \\ V_{\bar{z}}^a &= \Omega^a_b(z) \ell^b \\ M^{ab} &= \Omega^a_c(z) \Omega^b_d(z) M^{cd} \end{aligned} \quad (33)$$

with  $\Omega_b^a(z) \in O(3, 19)$ . After reduction, the two six-dimensional theories have the following field content:

$$\{g, B, \phi, V^A, M^{AB}\} \quad (34)$$

where the  $A, B$  indices refer to the  $O(4, 20)$  U-duality group. Here the 3-form  $C^{(3)}$  is dualized into a vector  $V_\bullet$  via  $dC^{(3)} = \frac{1}{4} * F_\bullet$ . The  $O(4, 20)$  index  $A$  and the  $O(3, 19)$  index  $a$  are related via  $A = \{\bullet, \bullet, a\}$  (see also appendix B). The 6D Lagrangians for the massive heterotic and iia actions read as

$$\begin{aligned} \mathcal{L}_{\text{het},6} &= \frac{1}{2} \sqrt{-g} e^{-2\phi} \left[ -R + 4|d\phi|^2 - \frac{3}{4} |\tilde{H}|^2 + \frac{1}{8} \text{Tr}(\mathcal{D}M\mathcal{D}M^{-1}) - |F^A|^2 \right. \\ &\quad \left. - \frac{1}{12} f_{ABC} f_{DEF} M^{AD} (M^{BE} M^{CF} - 3L^{BE} L^{CF}) \right] \\ \mathcal{L}_{\text{iia},6} &= \frac{1}{2} \sqrt{-g} e^{-2\phi} \left[ -R + 4|d\phi|^2 - \frac{3}{4} |H|^2 + \frac{1}{8} \text{Tr}(\mathcal{D}M\mathcal{D}M^{-1}) - e^{2\phi} |F^A|^2 \right. \\ &\quad \left. - \frac{1}{12} f_{ABC} f_{DEF} M^{AD} (M^{BE} M^{CF} - 3L^{BE} L^{CF}) \right] - \frac{1}{8} \epsilon B F_A F^A \end{aligned} \quad (35)$$

with the field strengths

$$\begin{aligned} H &= dB & \tilde{H} &= dB - V_A dV^A - \frac{1}{6} f_{ABC} V^A V^B V^C \\ F^A &= dV^A - \frac{1}{2} f_{BC}^A V^B V^C \\ \mathcal{D}M^{AB} &= dM^{AB} - f_{CD}^A V^C M^{BD} - f_{CD}^B V^C M^{AD}. \end{aligned} \quad (36)$$

The only non-zero structure coefficients are  $f_{\bullet ab} = m_{ab}$  with  $m_{ab}$  an anti-symmetric  $22 \times 22$  matrix and  $a, b$  indices of  $O(3, 19)$ . The Jacobi identity of  $f_{ABC}$  is identically satisfied for arbitrary  $m_{ab}$ . Both theories have an  $O(4, 20)$  U-duality pseudo-symmetry

$$V^A \rightarrow \Omega^A_B V^B \quad M^{AB} \rightarrow \Omega^A_C \Omega^B_D M^{CD} \quad f^{ABC} \rightarrow \Omega^A_D \Omega^B_E \Omega^C_F f^{DEF} \quad (37)$$

with  $\Omega_B^A \in O(4, 20)$ .

We are now in a position to establish an S-duality between the two reduced theories. The massless S-duality [4] maps the massless field strengths  $H$  and the Hodge dual  $\tilde{H}$  into each other. There is no obstruction to extending this to the massive case since both  $B$  and  $\tilde{B}$  are massless. The massive S-duality simply relates the massive field strengths to each other:

$$\begin{aligned} \underline{\text{h}} & \quad \underline{\text{iia}} \\ g_{\mu\nu} &= e^{-2\phi} g_{\mu\nu} \\ \phi &= -\phi \\ \tilde{H} &= e^{-2\phi} * H. \end{aligned} \quad (38)$$

with heterotic fields on the left-hand and iia fields on the right-hand side. These relations are obtained from the 7D duality relations (32) by the generalized reduction relations (57) of appendix B and (33). Thus they relate the two dual massive theories in six dimensions.

This finishes our discussion of the massive S-duality rules.

### 3.2. Brane solutions

The basic brane solutions of the heterotic theory are the  $hp$ -brane solutions with  $p = 0, 2, 4$ . Only the  $h4$ -brane solution requires a mass parameter. The  $h0$ -brane and  $h2$ -brane solutions are given by ( $p = 0, 2$ )

$$\begin{aligned} ds_{hp}^2 &= H^{p-2} [dt^2 - ds_p^2] - H^p dx_{5-p}^2 & e^{2\phi} &= H^{p-1} \\ F_{01\dots pm}^A &= \partial_m H^A & \mathcal{M}^{AB} &= \mathbb{I}^{AB} + 2H^{-1} H^A H^B. \end{aligned} \quad (39)$$

These are S-dual to  $dp$ -brane solutions on the iia side and can be obtained by applying the massless S-duality rules (25) to the iia d0- and d2-brane solutions (21). The h0-branes are reductions of the electric chiral null model  $F1//W1$ , which give (4, 4) charges, and H0-branes carrying (0, 16) electric charges with respect to the YM sector [24, 30]. The h2-branes are reductions of the magnetic chiral null model  $S5//KK5$ , giving (4, 4) charges, and the magnetic H6 branes, giving (0, 16) charges.

Upon applying the massive S-duality (27) to the d4-branes of the iia theory, we obtain  $\tilde{h}4$  domain wall solutions of the heterotic theory with  $m^A$  deformations (28). However, as explained in the previous subsection, we have not been able to find a higher dimensional origin of these massive deformations in the heterotic theory. Therefore, we do not give the explicit form of these solutions. By convention we will indicate solutions for which there is no viable higher dimensional origin with a tilde.

On the other hand, the mass deformations  $f_{ABC}$  of the heterotic theory, which do have a clear higher dimensional interpretation, give rise to the following h4-brane solutions:

$$ds_{h4}^2 = [dt^2 - ds_4^2] - H^2 dx^2 \quad e^{2\phi} = H^{-1} \quad \mathcal{M}_{ab} \neq 0. \quad (40)$$

We have not worked out which components of  $f_{ABC}$  support the h4 domain walls. These correspond to reductions of the magnetic chiral null model and the magnetic H6 branes. Since S-duality can be extended to mass deformations  $f_{ABC}$ , the h4-branes have S-dual partners in M-theory on  $K3 \times S^1$ : these are m4-branes with field configuration given by

$$ds_{m4}^2 = H [dt^2 - ds_4^2] - H^3 dx^2 \quad e^{2\phi} = H \quad \mathcal{M}_{ab} \neq 0. \quad (41)$$

Again, we have not worked out which components of  $f_{ABC}$  support the m4-brane solutions. In the next section, we will discuss how the m4-branes can be uplifted to the 11D  $KK6 \perp KK6$  intersections.

#### 4. Higher dimensional origin

We wish to consider the effect of S-duality on 1/2 BPS branes in 6D by examining their higher dimensional origin. For this purpose, it is useful to express the 6D S-duality in terms of 10D S- and T-dualities<sup>7</sup>:

$$\begin{array}{ll}
\text{IIA on } T^4/I_{6789} \equiv & \text{IIA on } T^4/I_{6789} \simeq K3 \\
\Downarrow T_6 & \\
\text{IIB on } T^4/(-)^{F_L} I_{6789} & \\
\Downarrow S & \Downarrow T_6 S T_6 \\
\text{IIB on } T^4/\Omega I_{6789} & \\
\Downarrow T_6 & \\
\text{IIA on } T^4/(-)^{F_L} I_{789} \Omega \equiv & \text{M-theory on } T^5/I_{78911} \simeq K3 \times S^1 \\
\Downarrow T_{789} & \\
\text{IIB on } T^4/\Omega & \Downarrow S T_{789} \\
\Downarrow S & \\
\text{IIB on } T^4/(-)^{F_L} \equiv & \text{Heterotic on } T^4
\end{array} \quad (42)$$

<sup>7</sup> Here we use that the different  $\mathbb{Z}_2$ -symmetries of string theory transform into each other under 10D S- and T-dualities [31]. For example:  $(-)^{F_L} = S\Omega S^{-1}$ . We consider the modding out with the  $\mathbb{Z}_2$ -symmetries only at the supergravity level. For a discussion of the twisted sector, see [32].

Here  $T_x$  ( $I_x$ ) denotes a T-duality (inversion) in the  $x$ -direction. Note that in the case of the IIA-theory (M-theory) the K3 manifold lies in the 6789 (78 911)-direction. In the latter case, we have used for this interpretation that  $(-)^{F_L} = I_{11} \Omega$ . From the figure above we see how the heterotic model compactified on  $T^4$  with enhanced gauge symmetry is related, via duality, to M-theory compactified on  $K3 \times S^1$  and Type IIA string theory on K3 in the singular orbifold limit. Thus, the 6D string theories are related by an  $S T_{789}$  duality between heterotic and M-theory and a  $T_6 S T_6$  duality between M-theory and the Type IIA superstring. These relations are conjectured for the full string theories, including the massive cases with background fluxes in the internal manifold.

At the supergravity level the relations correspond to the ordinary 6D duality relations in the massless case. However, we have seen that the supergravities can have different massive deformations, depending on the order of the SS reductions. So far, we found two classes of massive deformations: one with  $m^A$  and the other with  $f_{ABC}$ . Therefore, one does not expect that the massive deformations can be mapped onto each other by  $S T_{789}$  and  $T_6 S T_6$  at the level of the supergravity theory. We already concluded in section 3 that the mass deformations  $m^A$  and  $f_{ABC}$  do not have a higher-dimensional origin in *all* supergravities. In this section, we will analyse this higher dimensional origin further by applying string dualities to brane intersections.

For our present purpose it is sufficient to consider a truncation of the 6D theories. The full theories have a supergravity multiplet (with four gravi-photons) coupled to 20 vector multiplets. We consider the truncation to four vector multiplets. This corresponds to the reduction of  $D = 10$ ,  $N = 1$  supergravity on  $T^4$ , i.e. heterotic theory without Yang–Mills sector, IIA on  $T^4/I_{6789}$ , i.e. the untwisted sector of IIA string theory on K3 or M-theory on  $T^5/I_{78911}$ , i.e. the untwisted sector of M-theory on  $K3 \times S^1$ . From now on, we will always refer to these truncated theories which have an  $O(4, 4)$  duality symmetry.

We now apply the translation (42) of 6D duality to 10D duality to the brane solutions. We will use the lower-case letters h, m and d to denote solutions of heterotic theory on  $T^4$ , M-theory on  $K3 \times S^1$  and IIA on K3, respectively. We have summarized some of the nomenclature in the table below. From a 6D point of view, the iia and h theories with  $f_{ABC}$ -deformations are related via a S-duality. The two iia theories with  $m^A$ - and  $f_{ABC}$ -deformations cannot be related via a 6D supergravity relation (in the massless case they are related via an  $O(4, 20)$  rotation). The higher dimensional origin suggests a relation via a  $T_6 S T_6$  string duality but so far we are unable to realize this 10D string duality, after compactification, by a 6D supergravity duality. Note that not all components of  $f_{ABC}$  contain mass parameters. To be specific, we only have  $f_{\bullet ab} = m_{ab}$ .

In the remaining part of this section we will only consider solutions that preserve 1/2 supersymmetry. We therefore have to resort to intersections in the higher dimensional theory. We consider five different cases .

**Table 1.** The table indicates the massive deformations of the different 6D supergravity theories, their higher dimensional origin and the nomenclature for their brane solutions.

Theory	Deformation	Origin	Branes
iia	$m^A$	IIA on K3	dp
iia	$f_{ABC}$	M on $K3 \times S^1$	mp
h	$f_{ABC}$	H on $T^4$	hp

- (1) We first consider the 0-branes. We find that the S-duality in six dimensions relates, when uplifted to ten dimensions, the following 10D intersections<sup>8</sup>:

$$\begin{aligned}
 d0 &\leftrightarrow (0 \mid D2, D2): \left\{ \begin{array}{cccccccc} \times & - & - & - & - & \times & \times & - \\ \times & - & - & - & - & \times & \times & - \end{array} \right\} \\
 &\quad \updownarrow \\
 m0 &\leftrightarrow (0 \mid M2, M2): \left\{ \begin{array}{cccccccc} \times & - & - & - & - & \times & \times & - \\ \times & - & - & - & - & \times & \times & - \end{array} \right\} \\
 &\quad \updownarrow \\
 h0 &\leftrightarrow (1 \mid F1, W1): \left\{ \begin{array}{cccccccc} \times & - & - & - & - & - & - & \times \\ \times & - & - & - & - & - & - & \times \end{array} \right\}
 \end{aligned} \tag{43}$$

We find the electric chiral null model at the heterotic side. The duality between the electric chiral null model and intersections of D-branes was already noted in [24]. The  $D2 \perp D2$  intersections given above yield six charges while the other two charges come from  $D0//D4$ , which we do not give here. By considering the orbifold limit we can easily deduce the M-theory origin of the m0-branes. Six come from  $M2 \perp M2$  while the other two come from  $W1//M5$ . Note that since  $T_6 S T_6$  is a symmetry of massless IIA/M-theory compactified to six dimensions, the d0- and m0-branes constitute the same set of eight 0-branes and have the same 11-dimensional origin.

- (2) We next consider the 2-branes. Six of them arise as

$$\begin{aligned}
 d2 &\leftrightarrow (2 \mid D4, D4): \left\{ \begin{array}{cccccccc} \times & \times & \times & - & - & \times & \times & \times \\ \times & \times & \times & - & - & \times & \times & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 m2 &\leftrightarrow (3 \mid M5, M5): \left\{ \begin{array}{cccccccc} \times & \times & \times & - & - & \times & \times & \times \\ \times & \times & \times & - & - & \times & \times & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 h2 &\leftrightarrow (5 \mid S5, KK5): \left\{ \begin{array}{cccccccc} \times & \times & \times & - & - & \times & \times & \times \\ \times & \times & \times & - & - & \times & \times & \times \end{array} \right\}
 \end{aligned} \tag{44}$$

We find the magnetic chiral null model at the heterotic side [24, 34]. The other two 2-branes come from  $D2//D6$  and  $M2//KK6$ , respectively. The d2- and m2-branes are related by an element of the  $O(4, 4)$  duality symmetry group.

- (3) We now consider (some of) the 4-branes. They are, of course, the most interesting from the point of view of this paper. As we will demonstrate, not all domain walls have dual partners with a viable higher dimensional interpretation. In section 3, we derived a supergravity correspondence between the domain walls of heterotic theory on  $T^4$  and M-theory on  $K3 \times S^1$  with mass parameters contained in a 3-rank tensor  $f_{ABC}$ . This is the massive S-duality between the h4-brane and m4-brane which is diagrammatically described as follows:

$$\begin{aligned}
 \tilde{d}4 &\leftrightarrow (4 \mid KK6, KK6): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & z & \times & \times \\ \times & \times & \times & \times & - & z & \times & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 m4 &\leftrightarrow (4 \mid KK6, KK6): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & - & \times & \times \\ \times & \times & \times & \times & - & - & \times & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 h4 &\leftrightarrow (5 \mid S5, KK5): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & - & \times & \times \\ \times & \times & \times & \times & - & - & \times & \times \end{array} \right\}
 \end{aligned} \tag{45}$$

We have also indicated a  $\tilde{d}4$ -brane at the iia side. We have put a tilde to indicate that this brane has no viable higher dimensional interpretation. In the present case,  $\tilde{d}4$  is a solution of the IIA-theory compactified on  $K3$  with  $f_{ABC}$ -deformations whose  $D = 6$  supergravity formulation is unknown, in the sense that we do not know how to obtain such a theory from the SS reduction.

<sup>8</sup> Here the notation is as follows:  $\times$  corresponds to a worldvolume direction,  $-$  to a transverse direction and  $z$  to a Killing direction. The latter possibility corresponds to a  $R^p$  radius dependence of the effective tension with  $p > 1$  [33].

With the above 4-branes we can connect to all the mass parameters coming from axionic shift symmetries. In particular, the theories considered in section 3 obtained mass parameters using the U-duality in 7D. In the truncated case this is  $O(3, 3)$ , having six axions. On the heterotic side this corresponds to choosing the worldvolume direction of the heterotic magnetic chiral null model (with two charges) in one of the three directions of  $T^3$ . The 4-brane in 7D becomes a domain wall upon reduction on a circle and is therefore exactly the solution carried by  $f_{ABC}$ . On the M-theory side, the m4-branes correspond to two Kaluza–Klein monopoles wrapping different 2-cycles of the K3. Their isometries always lie in the K3 and not in the  $S^1$ . Finally, in terms of IIA on K3 the higher dimensional origin of the  $\tilde{d}4$ -branes lies in 10D KK6-branes. These 10D KK6-branes are the reductions of 11D KK6-branes in a transverse coordinate. We do not know how to use these 10D KK6-branes for the SS-reduction of supergravity, as was found in section 3.

- (4) We next consider the 1/2 BPS 4-brane corresponding to the mass deformations  $m^A$  of IIA on K3. Its higher dimensional origin is, as for the other dp-branes, an intersection of two D-branes. The  $D6 \perp D6$  gives rise to six masses (in the truncated theory) while the other two come from  $D4//D8$ . One should not expect to find an obvious interpretation of these deformations on the M-theory and heterotic sides, as we have learnt in section 3. Applying the 10D duality scheme to the first intersection we find

$$\begin{aligned}
 d4 &\leftrightarrow (4 \mid D6, D6): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & \times & \times & - \\ \times & \times & \times & \times & - & - & - & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 \tilde{m}4 &\leftrightarrow (4 \mid KK6, KK6): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & z & \times & - \\ \times & \times & \times & \times & - & z & - & \times \end{array} \right\} \quad (46) \\
 &\quad \updownarrow \\
 \tilde{h}4 &\leftrightarrow (4 \mid ??, ??): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & z & z & z \\ \times & \times & \times & \times & - & z & z & z \end{array} \right\}
 \end{aligned}$$

In terms of M-theory on  $K3 \times S^1$ , the  $\tilde{m}4$ -brane is an intersection of KK6-branes with their isometry in the  $S^1$ . Note that this would be an exotic brane in 7D, where its transverse space would be two dimensional including an isometry. We do not know how to generate the masses corresponding to the  $\tilde{m}4$ -branes in the SS reduction of M-theory on  $K3 \times S^1$ . Similarly, in terms of the heterotic theory compactified on  $T^4$  the higher dimensional interpretation is absent, in agreement with the supergravity findings of section 3.

- (5) The other 10D intersection reducing to a d4-brane is  $D4//D8$ , which gives

$$\begin{aligned}
 d4 &\leftrightarrow (4 \mid D4, D8): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & - & - & - \\ \times & \times & \times & \times & - & \times & \times & \times \end{array} \right\} \\
 &\quad \updownarrow \\
 \tilde{m}4 &\leftrightarrow (5 \mid M5, M9): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & \times & - & - \\ \times & \times & \times & \times & - & z & \times & \times \end{array} \right\} \quad (47) \\
 &\quad \updownarrow \\
 \tilde{h}4 &\leftrightarrow (4 \mid ??, ??): \left\{ \begin{array}{cccccccc} \times & \times & \times & \times & - & \times & z & z \\ \times & \times & \times & \times & - & z & z & z \end{array} \right\}
 \end{aligned}$$

In terms of M-theory on  $K3 \times S^1$ , the  $\tilde{m}4$ -brane is an M5//M9 intersection. The M9 brane is not a solution of the known 11D supergravity theory. On the heterotic side we cannot find a viable higher dimensional interpretation in terms of a known intersection, as indicated by the question marks.



One might expect that there are other heterotic theories in 6D by performing the Horava–Witten scenario [5] in a direction different from the 11th. One could, for example, consider M-theory on  $T^4 \times (S^1/\mathbb{Z}_2)$ . This is related to the heterotic theory on  $T^4$  by

$$\begin{aligned}
 \text{IIB on } T^4/(-)^{F_L} &\equiv & \text{H on } T^4 \\
 \Downarrow S & & \\
 \text{IIB on } T^4/\Omega & & \Downarrow S T_6 \\
 \Downarrow T_6 & & \\
 \text{IIA on } T^4/I_6\Omega &\equiv & \text{M-theory on } T^4 \times (S^1/\mathbb{Z}_2).
 \end{aligned} \tag{48}$$

By applying a  $S T_6$  transformation one can see that the massive deformations of the above two theories can be mapped onto each other<sup>9</sup>. It is impossible to relate these masses to the  $m_A$ -deformations of IIA on K3.

In summary, the analysis of the higher dimensional origin partially resolves the puzzle of massive S-duality. The h4- and m4-branes map onto each other under the massive S-duality map, as we also found in the supergravity approach. The S-duals of the d4-branes, on the other hand, do not have a viable 10D brane interpretation at the heterotic side. This analysis also tells us about the missing mass parameters in the theories. It is intriguing that both d4- and m4-branes and their  $T_6 S T_6$  duals correspond to  $KK6 \perp KK6$  intersections in 11D (apart from the M5//M9 intersection). Thus, one has a brane interpretation for both the  $m_A$  and  $f_{ABC}$  deformations in both theories. However, the corresponding mass parameter only appears in the 6D supergravity when the isometry of the KK monopole lies in the first of the two compactification manifolds. Explicitly, the isometry in the  $S^1$  corresponds to the  $m_A$  deformations and these are only found in IIA on K3 or M on  $S^1 \times K3$ . When the isometry lies in the K3 this corresponds to the  $f_{ABC}$ 's and these only occur in M-theory on  $K3 \times S^1$ .

## 5. Discussion and summary

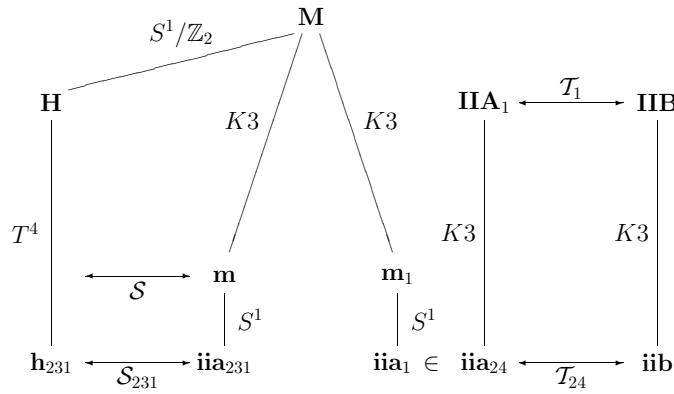
Using a generalized SS reduction scheme we have found two families of massive deformations in six-dimensional supergravities as illustrated in figure 2. In one case, the mass parameters are in the fundamental representation of the U-duality group  $O(4, 20)$ , in the other case they are in the 3-index antisymmetric representation of  $O(4, 20)$ . The T-duality between IIA and IIB on K3 can be extended with 24  $m_A$ -deformations while the S-duality between 11D supergravity on  $K3 \times S^1$  and the heterotic theory on  $T^4$  can be extended with 231  $f_{ABC}$ -deformations. Thus, in this paper, we have found that there are at least two cases where dualities between six-dimensional supergravities can include mass parameters.

In section 2, we have demonstrated the massive T-duality in six dimensions between Type IIA string theory compactified on K3 with background fluxes and Type IIB string theory compactified on K3.<sup>10</sup> Thus these theories are equivalent in five dimensions. In this paper, we used the simplest scenario where one includes background fluxes for 24 of the 100 axions in the 6D iib theory, and no background flux at the iia side. Instead, one can use the generalized reduction scheme of appendix B for the SS compactification of the iib theory from 6 to 5 dimensions and this induces more mass parameters [14]. A subset of these parameters is contained in the massive T-duality rules given in this paper.

As a by-product, see appendix B, we worked out the generalized toroidal SS reduction of the heterotic theory. At every step in this reduction we employ the full U-duality group. In this way, one obtains more mass parameters than if one only makes use of the axionic

<sup>9</sup> Up to a  $T_6$  transformation which acts as an element of  $O(4, 4)$  and therefore only rotates the  $f_{ABC}$ 's.

<sup>10</sup> One can reproduce one of the 24  $m_A$ 's from M-theory on  $K3 \times S^1$  as shown in figure 2 [35, 19]. This corresponds to the  $T_6 S T_6$ -relation of the D4- and M5-brane (47).



**Figure 2.** Web of massive dualities. The vertical lines are SS reductions and the subscripts are the number of massive deformations. Note that there are two families of massive deformations in six dimensions ( $24 m^A$ -deformations and  $231 f_{ABC}$ -deformations, see the text).

shift symmetries. In the case of a  $T^4$ -reduction the total number of mass parameters is  $120 + 153 + 190 + 231 = 694$ , which fit in a  $f_{ABC}$  multiplet and are constrained by the Jacobi identity. A subset of these,  $0 + 16 + 34 + 54 = 104$ , is generated by axionic shift symmetries and was derived in [15]. It remains to be checked whether our generalized SS reduction scheme gives rise to more independent mass parameters than those which follow from the axionic shift symmetries only or whether the new set of mass parameters is equivalent, up to a U-duality rotation, to that of [15] in  $D \leq 8$ . In section 3, we have derived the massive S-duality rules for the 231 parameters that arise in going from 7D to 6D. Thus, the massive S-duality in 6D is implied by the massless S-duality in 7D. The other massive deformations (463 for h and 24 for iia) seem to break the S-duality in six dimensions.

It is instructive to compare the  $D = 7$  to  $D = 6$  reduction with U-duality group  $O(3, 19)$  with the  $D = 10$  to  $D = 9$  reduction of IIB supergravity with U-duality group  $SL(2, R)$ . In the first case, we have 54 axionic shift symmetries while in the latter case there is a single shift symmetry. In the  $D = 10$  to  $D = 9$  reduction the SS reduction using the axionic shift symmetry relates a domain wall in  $D = 9$  to a D7-brane in  $D = 10$ . Similarly, we find that in the  $D = 7$  to  $D = 6$  SS reduction each of the 54 axionic shift symmetries relates a domain wall solution in  $D = 6$ , see (40), to a 4-brane solution in  $D = 7$ . The other SS reductions do not seem to relate in an obvious way brane solutions in  $D = 6(9)$  to brane solutions in  $D = 7(10)$ . This issue is under present investigation [36].

The  $f_{ABC}$ 's are the structure constants of a subgroup  $G \subset O(4, 20)$  in some non-standard basis [16, 17]. The  $f_{ABC}$ -deformations can also be obtained directly in 6D by gauging this subgroup [15]. We expect the same to be true for our general scheme. Since there are only 24 vectors available in the supergravity multiplet we expect that  $\text{rank}(G) \leq 24$ , though this is not manifest in the non-standard basis. It is of interest to investigate under which conditions a general massive deformation can be viewed as the result of a certain gauging in the supergravity theory and if so, which subgroup of the U-duality group is gauged.

Finally, it would be interesting to extend our work and consider compactifications to three dimensions. Massive supergravities in  $D = 3$  with neutral scalars were given in [37]. More recently, massive supergravities in  $D = 3$  with charged scalars have been constructed in [38, 39]. We expect that also in this case there exist different classes of massive supergravities with the mass parameters being in different representations of the U-duality group.

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## Appendix A. Notational conventions

Our notational conventions are similar to [4]: spacetime indices are omitted where possible but should be taken as fully antisymmetric and without factor. Thus  $H = dB - V_a dV^a$  implies  $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} - V_{a[\mu} \partial_{\nu} V_{\rho]}^a$  and

$$|H|^2 \equiv H_{\mu\nu\rho} H^{\mu\nu\rho} \quad |F^a|^2 \equiv F_{\mu\nu}^a M_{ab}^{-1} F^{b\mu\nu}. \quad (49)$$

The derivative  $d$  acts from the left. The Hodge star operator of a  $p$ -form in  $D$  dimensions is defined by

$$(\star H)^{\mu_1 \dots \mu_p} \equiv \frac{1}{p! \sqrt{-g}} \epsilon^{\mu_1 \dots \mu_D} H_{\mu_{D-p+1} \dots \mu_D}. \quad (50)$$

The different dimensional Levi-Civita symbols are related by  $\epsilon^{\mu_1 \dots \mu_{D-1} \bar{x}} \equiv \epsilon^{\mu_1 \dots \mu_{D-1}}$  when reducing over the coordinate  $x$ .

## Appendix B. Generalized SS reduction on a torus

In this appendix we will focus on the reduction of the heterotic theory. The compactification of the heterotic string with background fluxes was originally considered in [15]. More recently, such compactifications have been studied in [40]. Here we will rederive and extend the results of [15]. The results of this appendix are needed to construct the massive S-duality rules in section 3.

In general, supergravity theories acquire a U-duality symmetry group upon reduction. This U-duality group can be used to generate masses in further dimensionally reduced theories. For instance, consider the reduction on a torus, which is the product of circles. One can perform the compactification step-by-step, meaning circle-by-circle. At every step there is a U-duality group that can be used to give masses to the theory reduced further on the torus. There are restrictions however, which impose conditions on the mass parameters. Here we will describe this procedure for the heterotic theory but it can be applied to any theory with a U-duality symmetry group upon reduction.

Let us start with some general remarks on the U-duality symmetry groups that are considered in this paper. They are all of the form  $O(d, d+16)$  for some integer  $d$ . The  $O(d, d+16)$  index  $_A$  can be expressed in terms of the  $O(d-1, d+15)$  index  $_a$  as  $\{\bullet, \bullet, {}^a\}$ , while  ${}^A = \{\bullet, \bullet, {}^a\}$ . This iterative procedure stops at  $O(0, 16)$ , which has an index  $^k$  with  $k = 1, \dots, 16$ . All  $O(d, d+16)$  indices are raised or lowered with the metric

$$L_{AB} = L^{AB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & L_{ab} \end{pmatrix} \quad (51)$$

with  $L_{ab}$  the  $O(d-1, d+15)$  metric. The  $O(0, 16)$  metric is simply  $L_{kl} = (-\mathbb{I}_{16})_{kl}$ . The vectors of the theory form a fundamental representation  $V^A$  of the U-duality group. The scalars form a matrix  $M^{AB}$  iteratively defined by

$$M^{AB} = \begin{pmatrix} -e^{-2\sigma} + \ell_a \ell_b M^{ab} - \frac{1}{4} e^{2\sigma} \ell^4 & \frac{1}{2} e^{2\sigma} \ell^2 & \ell_a M^{ab} - \frac{1}{2} e^{2\sigma} \ell^2 \ell^b \\ \frac{1}{2} e^{2\sigma} \ell^2 & -e^{2\sigma} & e^{2\sigma} \ell^b \\ M^{ab} \ell_b - \frac{1}{2} e^{2\sigma} \ell^2 \ell^a & e^{2\sigma} \ell^a & M^{ab} - e^{2\sigma} \ell^a \ell^b \end{pmatrix} \quad (52)$$

where the scalars  $\ell_a$  are axionic ( $\ell^2 = \ell_a \ell^a$ ) and  $\sigma$  is a dilatonic scalar.  $M^{AB}$  parametrizes the coset  $O(d, d+16)/(O(d) \times O(d+16))$ , which contains  $d$  dilatons and  $d(d+15)$  axions. As any element of  $O(d, d+16)$  it satisfies  $M_{AD}^{-1} = L_{AB} M^{BC} L_{CD}$ . The coset  $O(0, 16)/(O(0) \times O(16))$  is empty:  $M^{kl} = (\mathbb{I}_{16})^{kl}$ . The U-duality symmetry acts as

$$V^A \rightarrow \Omega_B^A V^B \quad M^{AB} \rightarrow \Omega_C^A \Omega_D^B M^{CD} \quad (53)$$

with  $\Omega_B^A \in O(d, d+16)$ . This group consists of  $(16+2d)(15+2d)/2$  generators, of which  $d(d+15)$  shift the axions with a constant:  $\ell \rightarrow \ell + m$ .

Let us now specify to the heterotic theory. Upon reduction on a torus  $T^i$ , the heterotic theory obtains a global  $O(i, i+16)$  U-duality symmetry group. This symmetry group can be used to induce mass parameters in the lower dimensions. This has been done for a subset of the  $O(i, i+16)$  generators, namely those that induce the axionic shifts [15].

Using only these global symmetries for generalized reduction, this gives 0, 16, 16+34, 16+34+54, ... mass parameters in 9, 8, 7, 6, ... dimensions, respectively. However, there are constraints on these mass parameters. Since the masses break (part of) the U-duality group, one no longer can use all axionic shifts to generate masses in lower dimensions. Only U-duality transformations that leave the mass parameters invariant are true symmetries of the Lagrangian that can be used for the SS reduction. Thus the mass parameters have to satisfy certain product relations. We would like to generalize the result of [15] to the full U-duality group, i.e. use all possible generators to induce masses for the heterotic theory compactified on a torus. This can be done most conveniently step-by-step, i.e. by splitting the torus in a product of circles and using the U-duality group at every step [13].

Let us first quote the general result. Then we will iteratively prove that this indeed can be obtained from a generalized SS reduction. The massive heterotic action in  $(10-i)$  ( $i = 1, 2, 3, 4$ ) dimensions reads

$$\mathcal{L} = \frac{1}{2} \sqrt{-g} e^{-2\phi} \left[ -R + 4|d\phi|^2 - \frac{3}{4}|H|^2 + \frac{1}{8} \text{Tr}(\mathcal{D}M\mathcal{D}M^{-1}) - |F^A|^2 - \frac{1}{12} f_{ABC} f_{DEF} M^{AD} (M^{BE} M^{CF} - 3L^{BE} L^{CF}) \right] \quad (54)$$

with the field strengths

$$\begin{aligned} H &= dB - V_A dV^A - \frac{1}{6} f_{ABC} V^A V^B V^C \\ F^A &= dV^A - \frac{1}{2} f_{BC}^A V^B V^C \\ \mathcal{D}M^{AB} &= dM^{AB} - f_{CD}^A V^C M^{BD} - f_{CD}^B V^C M^{AD} \end{aligned} \quad (55)$$

where  $f_{ABC}$  ( $A$  is the  $O(i, i+16)$  index) are the fully antisymmetric expressions that contain the mass parameters (see below). Only in special cases of gauged supergravities can these constants be associated with the structure constants of a Lie algebra. The vectors  $V^A$  come from three different sources:  $V^h$  ( $h = 1, \dots, i$ ) are the Kaluza–Klein vectors,  $V_h$  stem from the 2-form while the  $V^k$  ( $k = 1, \dots, 16$ ) are the Yang–Mills vectors of the heterotic theory. The action (54) has a global  $O(i, i+16)$  U-duality pseudo-symmetry

$$V^A \rightarrow \Omega_B^A V^B \quad M^{AB} \rightarrow \Omega_C^A \Omega_D^B M^{CD} \quad f_{ABC} \rightarrow \Omega_D^A \Omega_E^B \Omega_F^C f_{DEF} \quad (56)$$

with  $\Omega^A_B \in O(i, i+16)$ . Only in the massless case, i.e.  $f_{ABC} = 0$ , it is a true symmetry acting only on fields. This subtlety will play an important role since only true symmetries can be used for a generalized SS reduction.

To explain the iterative derivation of (54) we start from the massless ten-dimensional heterotic action, i.e.  $i = 0$  and  $f_{abc} = 0$  (the gauge group will be considered to be broken to  $U(1)^{16}$ ). Conventional Kaluza–Klein reduction (with no dependence on the internal coordinates) gives the massless version of the reduced action, i.e. the action (54) with  $f_{AB}^C$  set equal to zero. We will now use the full U-duality symmetry to generate masses in lower dimensions. To see which structure coefficients can be made non-zero by generalized reduction we will iteratively perform the compactification step-by-step. Thus the action (54) is compactified on a circle, using the most general reduction relation consistent with the U-duality group. We employ the following reduction relations expressing the  $(10-j)$ -dimensional ( $j = i-1 = 0, 1, 2, 3$ ) fields in terms of the  $(9-j)$ -dimensional fields:

$$\begin{aligned}
 \frac{(10-j)D}{g_{zz}} &= \frac{(9-j)D}{-e^{-4\sigma/\sqrt{3}}} \\
 g_{zz} &= -e^{-4\sigma/\sqrt{3}} \\
 g_{z\mu} &= -e^{-4\sigma/\sqrt{3}} V_\mu^\bullet \\
 g_{\mu\nu} &= g_{\mu\nu} - e^{-4\sigma/\sqrt{3}} V_\mu^\bullet V_\nu^\bullet \\
 B_{\mu\nu} &= B_{\mu\nu} + V_{[\mu}^\bullet (V_{\nu]}^\bullet) + \ell^a V_{[\mu}^b V_{\nu]}^\bullet L_{ab} \\
 B_{z\mu} &= (V_\bullet)_\mu - \frac{1}{2} \ell^a V_\mu^b L_{ab} \\
 \phi &= \phi - \frac{1}{\sqrt{3}} \sigma \\
 V_\mu^a &= \Omega^a_b(z) (V_\mu^b + \ell^b V_\mu^\bullet) \\
 V_z^a &= \Omega^a_b(z) \ell^b \\
 M^{ab} &= \Omega^a_c(z) \Omega^b_d(z) M^{cd}
 \end{aligned} \tag{57}$$

with  $\Omega_b^a(z) \in O(j, j+16)$  ( $a = 1, \dots, 2j+16$ ) being the only  $z$ -dependence (the coordinate of the circle) on the right-hand side. These are the usual circle reduction formulae apart from the ‘reduction transformation’  $\Omega_b^a(z)$ . The single-valuedness of the reduced theory (independence of  $z$ ) follows from the U-duality symmetry (56) provided [13]

$$\Omega^{ab} \partial_z \Omega_{ac}(z) = - (m^{(j)})^b_c \tag{58}$$

with  $(m^{(j)})_{ab}$  an arbitrary antisymmetric  $(16+2j) \times (16+2j)$  matrix. From this it follows that

$$\Omega^a_b(z) = e^{- (m^{(j)})^a_b z}. \tag{59}$$

Reducing with these generalized relations we find the following field strengths for the vector fields

$$F^a = dV^a - (m^{(j)})^a_b V^\bullet V^b - \frac{1}{2} f_{bc}^a V^b V^c \quad F^\bullet = dV^\bullet \quad F_\bullet = dV_\bullet - \frac{1}{2} (m^{(j)})_{ab} V^a V^b \tag{60}$$

where  $V^\bullet$  is the Kaluza–Klein vector of the circle and  $V_\bullet$  comes from the 2-form  $B$ . These can be combined into  $F^A$  with  $A$  the  $O(i, i+16)$  index  $\{\bullet, \bullet, a\}$  where  $i = j+1$ . Thus we find the  $(10-i)$ -dimensional non-Abelian field strengths (55) with non-zero structure coefficients

$$f_{ABC}: f_{abc} \quad f_{ab\bullet} = (m^{(j)})_{ab}. \tag{61}$$

The scalar sector is somewhat more complicated but also reproduces (55) with the above structure coefficients.

There are restrictions to this generalized SS reduction scheme. If in the higher dimensional theory mass parameters are present, the U-duality symmetry group is (partly) broken. The only true symmetries are those that leave the structure coefficients invariant, i.e.

$$f_{abc} = \Omega_a^d \Omega_b^e \Omega_c^f f_{def}. \quad (62)$$

Thus not all U-duality transformations can be used to generate masses. Plugging in the ‘reduction transformation’ (59) one obtains relations between the mass parameters. These exactly boil down to the Jacobi identity for the structure coefficients of the reduced theory [15]:

$$f_{A[B}{}^C f_{DE]C} = 0. \quad (63)$$

In summary, there exists an iterative step-by-step prescription to obtain families of mass parameters in the heterotic theory compactified on a torus  $T^i$ . At the  $j$ th step ( $j = 0, 1, 2, i-1$ ) one can introduce a family  $m^{(j)}$  with  $(16 + 2j)(15 + 2j)/2$  independent mass parameters. These can be grouped into structure coefficients of the lower dimensional theory:

$$f_{ABC}: f_{abj} = (m^{(j)})_{ab} \quad (64)$$

where  $A = \{j, {}^j, {}_a\}$  is the index of the U-duality group of the reduced theory. When introducing more than one family of mass parameters one has restrictions

$$f_{A[B}{}^C f_{DE]C} = 0 \quad (65)$$

which follow from the fact that mass parameters break (part of) the U-duality symmetry group. For only one family of non-zero masses these are identically satisfied. Of course, it is possible to reshuffle the mass parameters by field redefinitions. In this way, one can redefine the structure coefficients by

$$f_{ABC} \rightarrow \Omega_A^D \Omega_B^E \Omega_C^F f_{DEF} \quad (66)$$

with  $\Omega_A^B$  an element of the U-duality group. Thus there is the issue of the number of independent parameters. Finally, our reduction scheme is more general than that of [15]. For instance, using only the axionic shift symmetries the heterotic theory cannot induce mass parameters in nine dimensions while using the full U-duality, as we do here, one can generate massive deformations already in nine dimensions.

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